Latency-aware VNF Chain Deployment with Efficient Resource Reuse at Network Edge

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Abstract—With the increasing demand of low-latency network services, mobile edge computing (MEC) emerges as a new paradigm, which provides server resources and processing capacities in close proximity to end users. Based on network function virtualization (NFV), network services can be flexibly provisioned as virtual network function (VNF) chains deployed at edge servers. However, due to the resource shortage at the network edge, how to efficiently deploy VNF chains with latency guarantees and resource efficiency remains as a challenging problem. In this work, we focus on jointly optimizing the resource utilization of both edge servers and physical links under the latency limitations. Specifically, we formulate the VNF chain deployment problem as a mixed integer linear programming (MILP) to minimize the total resource consumption. We design a novel two-stage latency-aware VNF deployment scheme: highlighted by a constrained depth-first search algorithm (CDFS) for selecting paths, and a path-based greedy algorithm (PGA) for assigning VNFs by reusing as many VNFs as possible. We demonstrate that our proposed algorithm can efficiently achieve a near-optimal solution with a theoretically-proved worst-case performance bound. Extensive simulation results show that the proposed algorithm outperforms three previous heuristic algorithms.

I. INTRODUCTION

Recent years, with the increasing popularity of mobile devices, service demands from mobile users grow exponentially. The emerging service requirements from applications such as virtual reality [1], Internet of Things [2] and wearable devices, are forming the majority of computation in the network [3]. Moreover, as the development of these applications, users have a strong desire for high-speed, low-latency (tens of milliseconds or less), always-on and multimedia-oriented connections [4], [5].

To accommodate this trend, telecommunication service providers introduce mobile edge computing (MEC), a new technology that provides an IT service environment and cloud-computing capabilities at the edge of the network, with close proximity to mobile users [6]. With this vision, the computation tasks can be offloaded from the core network to the edge, which is highly beneficial to reducing the end-to-end latency, ensuring service delivery, offering an improved user experience, and cutting down the resource utilization in the core network [6]. Meanwhile, a variety of devices such as base stations [7], enterprise gateways and home routers [8] are increasingly equipped with computation capabilities at the network edge, which makes MEC a hot research topic in both academia and industry [9], [10], [11], [12].

To provide flexible services with both reduced capital and operational expenses, another promising technology called network function virtualization (NFV) [13] has been widely advocated by service providers. In the context of MEC, network services can be flexibly provisioned using virtual network functions (VNFs) both in the back-end cloud (core network) and at the network edge, while serving the users in an on-demand fashion. Besides, service demands at the edge usually require rigorous low-latency in order to offer an instantaneous experience to mobile users. For instance, service demands from augmented reality require that the end-to-end latency should not exceed 10-20 ms [14]. As a typical application in NFV, a network service chain generally consists of an ordered sequence of VNFs. In practice, VNFs cannot only be shared by different service demands, but also can be deployed on different network devices. As a result, both computing and bandwidth resources can be efficiently utilized. Hence, in this work, we focus on an important problem: How to efficiently utilize the computing resources of edge devices while reducing bandwidth consumption of links, by taking the reuse of VNF instances and latency requirements into account?

However, it is very challenging to simultaneously achieve the above joint objectives since there exists a dilemma between the resource utilization of the servers and links. On one hand, in order to reserve as many bandwidth resources as possible, the service chain should be routed along the shortest path. As a result, some VNF instances located on other paths are too costly to be reused, thus new instances have to be launched along the shortest path to support the demands. On the other hand, if we focus on reusing as many launched VNFs as possible, a longer path might be selected for the service chain with more bandwidth resources consumed, and a higher chance to violate the latency limitations. For example, a service chain
Towards the network edge, we present an optimization formulation for the VNF chain deployment problem that minimizes the resource consumption of both servers and links with the latency guarantees. Particularly, we take the queuing delay at the APs into account for meeting the latency requirements.

We seek to find a deployment scheme for service chains in sequence and divide the original problem into two subproblems, i.e., selecting paths first and then assigning VNFs, and propose a two-phase near-optimal algorithm to efficiently solve the problem.

We provide theoretical analysis for the proposed algorithm with a worst-case performance bound. Extensive simulations based on real-world topologies demonstrate that our proposed algorithm can efficiently and effectively obtain a solution, and outperform three existing heuristic algorithms in terms of resource consumption.

II. RELATED WORK

With the increasing popularity of NFV, the traditional service provision problem within the telecommunications industry has been migrated to the NFV scenario. In short, there is a continuous effort to investigate the problem from different angles [4], [18]–[23]. Among them, [18]–[21] tackle the VNF placement and routing problem, which is a significant problem arisen from NFV.

A. VNF Placement and Routing in General

Most existing works focus on the VNF placement and routing problem with different objectives and propose corresponding solutions. For example, Fei et al. [24] investigate the problem for VNF providers to dynamically scale the VNF deployments and reroute traffic demands. They first employ an online learning approach to predict the arriving traffic flows and then propose a joint online algorithm for new VNF instance assignment and service chain rerouting. Sang et al. [20] tackle the VNF placement problem to minimize the total number of VNF instances in a network reduced from the set cover problem. They design two greedy algorithms to solve the problem, both of which are proven to be asymptotically optimal.

There are also some works targeting on simultaneously optimizing the consumption of both the computing and bandwidth resources. Cohen et al. [21] address the VNF placement problem within physical networks, which aims at minimizing the total setup costs of functions and the distance costs between the clients and the servers. They provide a near-optimal approximation algorithm with theoretically proven performance. However, they assume that each demand of service only requires a single VNF rather than a service chain. In [18], the authors discover the resources relation between links and servers, and study the joint problem of VNF placement and path selection to better utilize both the network and server resources, whose objective is to maximize the total size of admitted demands rather than optimizing resource utilization, and they ignore the service latency requirements.

B. VNF Placement and Routing at the Network Edge

The above works all focus on a cloud scenario, which cannot be directly applied to the network edge, because of the characteristics of the network edge, such as resource shortage and the multiple access points for users. There also exist a few works tackling the VNF placement (and routing) problem at the network edge so far. For example, [25] studies the
TABLE I
KEY NOTATIONS.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( G = (U, D, \mathcal{E}) )</td>
<td>The graph of the network</td>
</tr>
<tr>
<td>( S )</td>
<td>The set of VNF service chains, ( S_i \in S ) is the service chain from user ( u_i ) in ( U )</td>
</tr>
<tr>
<td>( e(u,v) )</td>
<td>The available bandwidth resource on link ((u,v))</td>
</tr>
<tr>
<td>( B_{i}(u,v) )</td>
<td>The bandwidth capacity of link ( e(u,v) \in \mathcal{E} )</td>
</tr>
<tr>
<td>( t_{i}(u,v) )</td>
<td>The latency of the link ( e(u,v) ) when ( S_i ) using it</td>
</tr>
<tr>
<td>( C_{j} )</td>
<td>The resource capacity of hardware device ( d_j \in D )</td>
</tr>
<tr>
<td>( R_{n} )</td>
<td>The resource demand of a type-( n ) VNF instance</td>
</tr>
<tr>
<td>( W_{n} )</td>
<td>The processing capacity of a type-( n ) VNF instance</td>
</tr>
<tr>
<td>( \beta_{l} )</td>
<td>The maximum latency constraint of ( S_{l} \in S )</td>
</tr>
<tr>
<td>( (\phi_{l}, t_{l}, r_{l}) )</td>
<td>The three-tuple of service chain ( S_{l} ); ( \phi_{l} \in D ) is the set of available access points of ( S_{l} ); ( t_{l} \in d_{j} ) denotes the destination of ( S_{l} ), and ( r_{l} ) is the flow rate of ( S_{l} )</td>
</tr>
<tr>
<td>( X_{i,j}^{l} )</td>
<td>Whether the ( l^{th} ) VNF of service chain ( S_{i} ) is served by server ( d_{j} )</td>
</tr>
<tr>
<td>( Z_{i}^{l}(u,v) )</td>
<td>Whether the subchain between the ( l^{th} ) and ( l^{th} ) VNF of ( S_{i} ) is routed on link ( e(u,v) )</td>
</tr>
<tr>
<td>( T_{i,n} )</td>
<td>Whether the ( l^{th} ) VNF of the service chain ( S_{i} \in S ) requires the type-( n ) VNF</td>
</tr>
<tr>
<td>( Y_{j,n} )</td>
<td>The number of type-( n ) VNF instances in server ( d_{j} )</td>
</tr>
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</table>

problem of jointly optimizing the access network selection and service placement problem for MEC. They design an online framework to improve the QoS of the services, but each service only contains a single VNF. Zhang et al. [26] focus on the VNF placement problem in 5G service-customized network slices, they propose a general 5G network slice framework, taking the VNF interference into account, and propose an adaptive interference-aware heuristic (AI) approach to automatically place VNFs in each 5G network slice. However, they do not consider the resource optimization at all. Cziva et al. [27] follow the work [4] to investigate the latency optimization of single VNF placement and routing problem without considering the reusability of the VNF instances. In addition, they emphasize the limited resource provision at the network edge but do not take the resource optimization into consideration.

To the best of our knowledge, the efficient VNF chain deployment problem for resource optimization with latency limitations at the network edge has not been investigated before. Hence, in this paper, our objective is to simultaneously reuse the server resources and reduce the bandwidth consumption while providing latency guarantees.

### III. MODEL AND FORMULATION

#### A. The Network Model

We model a network as an undirected graph \( G = (U, D, \mathcal{E}) \), where \( U, D \) and \( \mathcal{E} \) denote the set of users, the set of hardware servers (including all kinds of devices equipped with computation capabilities in the network edge, such as the access points, the switches, the routers as shown in Fig. 1), and the set of physical links between the servers, respectively. We assume each server \( d_j \in D \) has a resource capacity denoted by \( C_j \), i.e., the amount of available resources in terms of CPU and memory. Similarly, each link \( e(u,v) \in \mathcal{E} \) between server \( d_u \in D \) and server \( d_v \in D \) has a bandwidth capacity denoted by \( B(u,v) \).

Say there are different service demands from different users, and each of them requires a service chain consisting of an ordered sequence of VNFs. We use \( N \) and \( S \) to denote the set of different types of VNFs and the set of different service chains, respectively. For each user \( u_i \in U \), we denote \( S_i \in S \) as the requested service chain, and \( |S_i| \) is the length (the number of the VNFs) of \( S_i \). We next assume that \( S_i \in S \) is the \( l^{th} \) VNF of service chain \( S_i \), and \( \Phi_{i}^{l} \) is the subchain between the \( S_{i}^{l} \) and \( S_{i}^{l'} \) (\( 1 \leq l \leq l' \leq |S_{i}| \), \( \Phi_{i}^{l'} \subseteq S_{i} \)). Specifically, we introduce \( T_{i,n} \) to indicate whether the \( l^{th} \) VNF of the service chain \( S_i \) is type-\( n \) \( \in \mathbb{N} \). In reality, a user might require more than one service chain, which can be regarded as different users in the same physical location require different service chains.

We further define a three-tuple \( (\phi_{l}, t_{l}, r_{l}) \) for service chain \( S_i \) to indicate the source, the destination, and the flow rate of \( S_{i} \), respectively. Specifically, \( \phi_{l} \) is the set of the nearby APs collocated to different edge clouds, which are in close proximity to \( u_i \), which means \( S_{i} \) can access the network from the \( k^{th} \) AP \( d_{i,k} \in \phi_{l} \). Since each \( d_{i,k} \in \phi_{l} \) corresponds to a server \( d_{j} \in D \), such as the base station as shown in Fig. 1, we simply denote by \( \alpha_{j} \) the processing capacity of AP \( d_{j} \in D \). Here, we assume that the flow rate of a service chain always remains unchanged when traversing different servers, since traffic change effects have been investigated in [28], and our formulation can easily extend to the changing case.

Lastly, let \( \mathbb{P} \) be the set of the physical paths on which service chains are routed along. Each \( P_{i} \in \mathbb{P} \) consists of an ordered sequence of physical links for service chain \( S_{i} \). For instance, \( \mathbb{P}_{i} = \{e(d_{1},d_{2}),e(d_{2},d_{3}),e(d_{3},d_{4})\} \) in Fig. 1 indicates that the routing path of the service chain \( S_{i} \) contains four server nodes \( d_{1}, d_{2}, d_{3} \) and \( d_{4} \).

#### B. Problem Formulation

We assume all the VNFs can be shared by different service chains so that their residual processing capacities can be maximally reused. We denote by \( Y_{j,n} \), the number of type-\( n \in \mathbb{N} \) VNF instance in server \( d_{j} \in D \). Each time when a service chain requires a type-\( n \) VNF from \( d_{j} \), if there is reusable type-\( n \) VNF instance, \( Y_{j,n} \) will remain unchanged; else we will launch a new type-\( n \) instance in \( d_{j} \), and \( Y_{j,n} \) becomes \( Y_{j,n} + 1 \). When launching a type-\( n \) VNF instance, it requires \( R_{n} \) device resource. To guarantee that the total resource consumption of each server \( d_{j} \) should not exceed the resource capacity \( C_{j} \), we have:

\[
\sum_{n \in \mathbb{N}} Y_{j,n} R_{n} \leq C_{j}, \quad \forall d_{j} \in D. \tag{1}
\]

We use \( Z_{i}^{l}(u,v) \) to represent whether the subchain \( \Phi_{i}^{l} \) of \( S_{i} \) traverses link \( e(u,v) \). In the same way, the sum of throughput of all the traffic flows along link \( e(u,v) \in \mathcal{E} \) should not exceed...
its bandwidth capacity \( B(u, v) \), which can be represented as follows:

\[
\sum_{S_i \in S} \sum_{\phi_i \subseteq S_i} Z_{i(u,v)}^{\prime l} \cdot r_i \leq B(u, v),
\]

\( \forall e(u, v) \in E, 1 \leq l < l' \leq |S_i| \). \hspace{1cm} (2)

Let \( W_n \) be the processing capacity of each type-\( n \) VNF instance. To ensure that all the VNF instances on any server \( d_j \) have sufficient capacities to process the flows, we have:

\[
\sum_{S_i \in S} \sum_{\phi_i \subseteq S_i} T_{i,n}^{l} \cdot X_{i,j}^l \cdot r_i \leq Y_{j,n} \cdot W_n, \forall n \in N, \forall d_j \in D,
\]

\( \sum_{S_i \in S} \sum_{\phi_i \subseteq S_i} T_{i,n}^{l} \cdot X_{i,j}^l \cdot r_i \leq Y_{j,n} \cdot W_n, \forall n \in N, \forall d_j \in D, \)

\( \forall e(u, v) \in E, 1 \leq l < l' \leq |S_i| \). \hspace{1cm} (3)

where \( X_{i,j}^l \) denotes whether the \( i \)th VNF of service chain \( S_i \) is served by server \( d_j \).

As mentioned above, we assume that each service chain \( S_i \) carries a single flow, i.e., each \( S_i \) just gets served by one server. Then, we have the following constraint for \( X_{i,j}^l \):

\[
\sum_{d_j \in D} X_{i,j}^l = 1, \quad \forall S_i \in S, \forall l \in [1, |S_i|].
\]

\( \sum_{d_j \in D} X_{i,j}^l = 1, \quad \forall S_i \in S, \forall l \in [1, |S_i|]. \hspace{1cm} (4) \)

Next, for each service chain \( S_i \), if it can be successfully accepted, we should make sure the total latency will not exceed its latency limitation \( \beta_i \). Here, we consider two kinds of latency, namely the queuing delay in the AP \( d_j \notin \phi_i \) selected by \( S_i \), and the propagation delay and transmission delay \( l_{i(u,v)} \) on the link \( e_{(u,v)} \). To analyze the queuing delay of users at the AP, we model each AP as an M/M/1 queue. By applying the Little’s law, the average queuing delay in \( d_j \) can be calculated as follows:

\[
q_j = \frac{1}{\alpha_j} - \sum_{S_i \in S} A_{i,j} \cdot r_i,
\]

\( q_j = \frac{1}{\alpha_j} - \sum_{S_i \in S} A_{i,j} \cdot r_i \hspace{1cm} (5) \)

where \( A_{i,j} \) denotes whether the service chain \( S_i \) chooses the AP \( d_j \notin \phi_i \) to access the edge network. Then we have:

\[
\sum_{d_j \in D} A_{i,j} \cdot q_j + \sum_{\phi_i \subseteq S_i} Z_{i(u,v)}^{\prime l} \cdot l_{i(u,v)} \leq \beta_i,
\]

\( \forall e(u, v) \in E, \forall S_i \in S, 1 \leq l < l' \leq |S_i|, j = u \) if \( l = 1 \).

\( \forall e(u, v) \in E, \forall S_i \in S, 1 \leq l < l' \leq |S_i|, j = u \) if \( l = 1 \).

Since \( S_i \) can only choose one AP each time, we have the following constraint for \( A_{i,j} \):

\[
\sum_{d_j \in D} A_{i,j} = 1, \quad \forall S_i \in S.
\]

\( \sum_{d_j \in D} A_{i,j} = 1, \quad \forall S_i \in S \hspace{1cm} (7) \)

Last but not least, we should guarantee all the VNFs in \( S_i \) will be served, which can be represented as follows:

\[
\sum_{S_i \in S} X_{i,j}^l = |S_i| \quad \forall S_i \in S.
\]

\( \sum_{S_i \in S} X_{i,j}^l = |S_i| \quad \forall S_i \in S \hspace{1cm} (8) \)

Our objective is to jointly minimize the resource consumption on servers and links. Since the device resource consumption at server \( d_j \) is \( \sum_{n \in N} Y_{j,n} \cdot R_n \), and the total bandwidth consumption of the link \( e(u,v) \) is \( \sum_{S_i \in S} \sum_{\phi_i \subseteq S} Z_{i(u,v)}^{\prime l} \cdot r_i \).

Then the optimization problem of the joint resource consumption can be formulated as follows:

\[
\min \sum_{d_j \in D, n \in N} Y_{j,n} \cdot R_n + \sum_{e(u,v) \in E} \sum_{S_i \in S} \sum_{\phi_i \subseteq S_i} Z_{i(u,v)}^{\prime l} \cdot r_i
\]

s.t. \( (1), (2), (3), (4), (5), (6), (7), (8) \)

(9)

**IV. TWO-PHASE NEAR-OPTIMAL SOLUTION**

**A. Problem Complexity**

The VNF deployment problem has been proved to be NP-hard in many previous works [17], [29]–[32]. Here, we relax the formulation of our VNF chain deployment problem exhibited in Eq. (9) with only server resource constraints defined in Eq. (1), Eq. (3) and Eq. (4), and VNF constraint in Eq. (8), which then becomes a reduction form of a classical bin packing problem. Many existing works have proved the NP-hardness of the bin packing problem, such as [33]–[35]. Note that with additional constraints in Eq. (2), Eq. (5), Eq. (6), Eq. (7), we can conclude that the VNF chain deployment problem in Eq. (9) is also NP-hard.

**B. An Overview of Solution**

Since the VNF deployment problem is proved to be NP-hard, it is computationally expensive to derive the optimal solution for deploying all service chains. By making an assumption that a sequence of service chains arrive into the system one by one without the knowledge of future arrivals, our problem focuses on how to deploy and route a single service chain to minimize the resource consumption.

However, it is still tricky to solve the problem. Thus, we seek to divide the original single chain problem into two subproblems, i.e., selecting paths first and then assigning VNFs. Specifically, in the first subproblem, for service chain \( S_i = (\phi_i, t_i, r_i) \), we find all the feasible paths from source \( \phi_i \) to destination \( t_i \) which are called pre-routing paths, under the constraints of the bandwidth of links and the latency of the service chain. We then put the paths into a set of paths denoted by \( P_i^A \). After that, to solve the second subproblem, we take turns to pick the paths out of \( P_i^A \). For each \( p \in P_i^A \), considering the reusable VNF instances and the residual resource capacities of the servers, we assign VNFs along the path with the maximum reusing number. Next, we choose an assignment scheme with minimum resource consumption among all the paths. This ensures our solution obtained by the two-phase strategy for single chain is near-optimal. In the end, we add up all the resource consumption of single chains to get the total resource consumption of all the arriving service chains.

**C. Selecting Paths**

For the first subproblem, we construct a new weighted undirected graph \( G' = (D, E; w_1, w_2) \) (see Fig. 2(a)), where

![Fig. 2. Reconstruct a new graph H according to the repeating times of the links. In (a), the numbers outside parentheses denote the repeating times, the numbers inside parentheses denote the latency of the links. The numbers in (b) denote the latency of the links.](image-url)
$w_1$ and $w_2$ denote the delay and residual bandwidth of each link, respectively. In other words, given a graph $G'$ and a certain service chain $S_i$ with latency limitations, our objective is to find all the feasible paths in $G'$, which consist of a sequential set of links from $φ_i$ to $t_i$ without exceeding the bandwidth capacity limitations.

**Reconstruct a new network graph.** To describe the available servers and links for service chain $S_i = \{s_1, s_2, ..., s_t\}$, we construct a new weighted mixed multi-graph $H = (D', E'; w)$ (see Fig. 2(b)) from graph $G'$, where the weighted value of link $e(u, v)$ is $l_i(u, v)$. Hereinafter, we detail the construction of graph $H$ in three steps:

**Step 1: Servers deriving.** Firstly, we add all the available APs $d_j \in φ_i$ of $S_i$ to $H$. We then assume that the minimum resource consumption among all the VNF $s_i \in S_i(l \in \{1, ..., |S_i|\})$ instance is $R_{\text{min}}^i$. If the residual resource on server $d_j \in D$ is more than $R_{\text{min}}^i$, or there are any available VNF instances could be re-used by VNF $s_i \in S_i(l \in \{1, ..., |S_i|\})$, then we add server $d_j$ to $H$.

**Step 2: Links deriving.** There are chances that a VNF of a chain will be served by a server where the upstream VNF has been served, as long as the server has sufficient resources left, which means there might exist loops in the routing paths. Meanwhile, if the bandwidth capacity is sufficient, each link of the graph could be traversed multiple times. For example, given $S_i = (d_0, \gamma, d_1) = \{s_0, s_1, s_2, s_3\}$ and a part of network $G'$ (see Fig. 3(a)), if we deploy $s_0$, $s_1$, $s_2$ and $s_3$ on server $d_1$, $d_2$, $d_3$ and $d_2$, respectively, the traffic flow will pass through the link $(d_2, d_3)$ twice, considering the links in $G'$ are undirected.

Since our objective is to minimize resource consumption with latency constraints at the network edge, it is unreasonable for a service chain to pass through a link time and again. Thus, for each available link, we will continue to use CDFSA to select the feasible paths. We then employ a recursive traversal algorithm called Depth First Search Algorithm (DFS) to select the feasible paths.

![Diagram](diagram.png) Fig. 3. The solid lines and the dotted lines in (a) denote the physical links and the flow of service chain $S_i$, respectively. The data in (b) indicates the reusable VNF instances for $S_i$ at each server, which is derived from Eq. (11).

**Algorithm 1: A Constrained Depth First Search Algorithm (CDFSA)**

**Input:** $H(\phi_i, r_i, t_i)$

**Output:** $P^\Lambda_i$

1. Initialize the adjacency matrix $M$ and weighted value matrix $E$;
2. Initialize a stack called $c_p = \emptyset$ to store the nodes of the path.
3. for each $d_j \in \phi_i$ do
   4. $u \leftarrow d_j$;
   5. Calculating the queuing delay $q_j$ of $d_j$ as Eq. (5) $L_t \leftarrow q_j$;
   6. function $\text{Selection\_Paths}(u, t_1, L_t, \beta_t)$ push the current node $u$ into stack $c_p$.
      if the current node $u = \text{end point} t_i$ then
         Fetch the path from $c_p$ and put it into the set $P^\Lambda_i$.
         return;
      end
   7. for each node $v$ in graph $H$ do
      8. if $M[u][v] > 0$ & $L_t + E[u][v] < \beta_t$ then
         9. $M[u][v] \leftarrow 0$;
      end
      else if $M[u][v] = 1$ then
         10. $M[u][v] \leftarrow 0$;
      end
   11. end
   12. $\text{Selection\_Paths}(v, t_i, L_t + E[u][v], \beta_t)$;
   13. end
   14. end
   15. end
   16. end
   17. end
   18. end
   19. end
   20. end
   21. end
   22. end
   23. end
   24. end
   25. end
   26. end

In addition, we also use another weighted value matrix $E$ to describe the latency of the links, where each element $E[u][v]$ in $E$ denotes the latency of link $e(u, v)$ as $l_i(u, v)$.

We then employ a recursive traversal algorithm called Depth First Search Algorithm (DFS) to select the feasible paths. Specifically, we first choose one of the AP $d_j \in \phi_i$ and add the queuing delay on AP $d_j$ to the sum of the latency. Considering the current server is $u$, for each available link $e(u, v)$, we add it to the path as long as the sum of the latency is still under limitation. If link $e(u, v)$ is successfully added to the path, we then should update the adjacency matrix $M$ accordingly. In the case of $M[u][v] = 1$, i.e., the link $e(u, v)$ is undirected and can only be traversed once, we update $M[u][v] = 0$ and $M[v][u] = 0$, which means the $e(u, v)$ and $e(v, u)$ will be removed from graph $H$. In another case of $M[u][v] = 2$,
we removed directed link \((u, v)\) from the graph by setting \(M[u][v] = 0\), and keep the directed link \((v, u)\) unchanged. By recursively calling the function Selection Paths in Algorithm 1 until reaching node \(t_i\), we can obtain the set of pre-routing paths \(P_i^k\) for \(S_i\). In the end, if there does not exist any feasible paths from \(d_j \in \phi_i\) to \(t_i\), the service chain \(S_i\) will be rejected.

### D. Assigning VNFs
Given a single service chain \(S_i\) and a set of pre-routing paths \(P_i^k\), our next objective is to choose an assignment scheme with the minimum resource consumption. Since we cannot foresee which path will yield the optimal solution, we take turns to pick the pre-routing path \(p\) out of \(P_i^k\) and compute a local optimal assignment scheme for each \(p\). Finally, by comparing all the local optimal schemes, we choose the globally optimal scheme among all the pre-routing paths.

For a selected pre-routing path \(p \in P_i^k\), let \(k\) be the length (the number of links) of the path, the corresponding number of servers along the path is \(k + 1\). Hence, we can define path \(p\) as a sequence of server nodes \(p = \{s_{p_0}, s_{p_1}, \ldots, s_{p_n}\}\) \((s_{p_i} \in \mathbb{D}, i \in \{0, 1, \ldots, k\})\). Since there might loop in path \(p\), different server nodes \(s_{p_i}, s_{p_j} \in p\) might correspond to the same server \(d_j \in \mathbb{D}\). For each server \(d_j \in \mathbb{D}\), we should make sure that the resource consumption of all VNFs in \(d_j\) should not exceed its resource capacity, and the processing capacities of all VNFs in \(d_j\) have sufficient capacities to process the flows. Besides, it is essential to preserve the ordered sequence of the VNFs in the service chain. Unfortunately, the assignment subproblem can be reduced as a bin-packing problem, which is still an NP-hard problem.

#### A Path-based Greedy Algorithm (PGA).
To solve the problem, we propose a near-optimal algorithm called PGA. The key idea of PGA is to sequentially assign VNFs of \(S_i\) to the nodes along \(p\) while reusing as many available VNF instances as possible, with the constraints of the server resources and VNF processing capacities.

To begin with, we explain how to compute the maximum number of reusable VNF instances. Given a service chain \(S_i = \{s_0, s_1, \ldots, s_{|S_i| - 1}\}\) and a pre-routing path \(p = \{s_{p_0}, s_{p_1}, \ldots, s_{p_k}\}\) as inputs, we first construct a new \(|S_i| \times (k + 1)\) matrix \(A\) to describe the relationship between \(S_i\) and \(p\). Each element \(A[i][j] \in A\) indicates whether \(s_i \in S_i\) could reuse the VNF instance at server node \(s_{p_j}\). We denote by \(W_{j,u}\) the residual type-\(n\) VNF instance processing capacity of device \(d_j\), which can be obtained from Sec. III. Assuming the \(s_i\) is type-\(n_i\) and the server node \(s_{p_j}\) corresponds to server \(d_j\), then \(A[i][j]\) can be calculated as follows:

\[
A[i][j] = \begin{cases} 
1 & \text{if } W_{(d_j), (s_i)} - r_i \geq 0 \\
0 & \text{otherwise.} 
\end{cases} \tag{11}
\]

For example, for the flow of Fig. 3(a) and the VNF capacity as shown in Fig. 3(b), we have:

\[
A = \begin{pmatrix}
\begin{array}{cccccc}
d_0 & d_1 & d_2 & d_3 & d_2 & d_4 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
\end{array}
\end{pmatrix}
\tag{12}
\]

Besides, if VNF \(s_i \in S_i\) requires a new instance, we assume its resource consumption is \(R_i\).

It is vital to keep the order of the VNFs. For example, if we deploy \(s_i \in S_i\) to \(s_{p_j}\), then \(s_{i}' \in S_i'(l' > l)\) can only be deployed at \(s_{p_j'}(j' \geq j)\), as shown in Fig. 4. We employ a recursive function assignment in Algorithm 2 to illustrate the operations in detail. For each \(s_i \in S_i\), there are two types of servers where \(s_i\) can be assigned: the servers with reusable VNF instance denoted as type-\(R\) and the server without reusable VNF instance but with enough resource to launch at least a new VNF instance denoted as type-\(V\). And there are four cases in each level of the recursion: 1) first find the type-\(R\) and terminate the loop for \(s_i\), this would be
the best assignment position for \( s_l \); 2) we first find the type-\( V \) and then the type-\( R \) in sequence; 3) we only find the type-\( V \); 4) we find neither types, i.e., nowhere \( s_l \) can be assigned, thus service chain \( S_l \) will be rejected. Note that for each level of the recursion, we only need to try at most two optional nodes for \( s_l \) assignment. Hence, for each \( s_l \), we call the recursive function up to two times, which significantly reduces the computation complexity of PGA.

For each pre-routing path \( p \in \mathcal{P}_i \), we can only get a local optimal assignment solution. To get the globally optimal solution, we first compute the total resource consumption of \( p \) by adding the minimum server resource consumption and the bandwidth consumption (the product of the length of \( p \) and flow rate of \( S_i \)) together. We then compare the total resource consumption of each pre-routing path and choose the globally optimal solution as our final result. In this way, the solution obtained by the two-stage strategy is still optimal.

E. The Worst-Case Performance Bound

However, the optimal solution obtained by PGA is not optimal for the VNF chain deployment problem, since we aim at finding a deployment scheme with maximum reusing number and then calculating the minimum resource consumption, rather than directly deriving minimum resource consumption of the problem. For example, for \( S_i = \{s_1, s_2, s_3, s_4\} \), we get an “optimal” solution \( A \) through PGA: if there are only reusable VNF instances for \( s_1, s_2 \) and \( s_3 \), then the maximum reusing number will be 3 and the server resource consumption will be \( R_3^s \). In another case, say there’s a solution \( B \), where \( s_3, s_4 \) can reuse the existing VNFs. Then the reusing number of \( B \) is 2, and the server resource consumption is \( R_2^s \). In the case of \( R_1^s + R_2^s < R_3^s \), solution \( B \) is superior to our PGA solution \( A \). That is to say, we obtain a near-optimal solution for all the pre-routing paths in \( \mathcal{P}_i \) by PGA. Accordingly, our two-stage algorithm is near-optimal. Now, we compute the worst-case performance bound of the two-stage solution in detail.

Specifically, given a service chain \( S_i \) and a pre-routing path \( p \in \mathcal{P}_i \), we have obtained the optimal assignment scheme with maximum reusing number \( p_r \) and minimum server resource consumption \( C_s \). Then the minimum total resource consumption \( C \) among all the pre-routing paths \( p \in \mathcal{P}_i \) (with a length denoted as \( k \)) can be calculated as:

\[
C = \min\{C_s + r_i \cdot k, \forall p \in \mathcal{P}_i \}.
\]

Lemma 1. For a service chain \( S_i \) and a same pre-routing path, assume that the reusing number of our solution and the optimal solution are \( p_r^A \) and \( p_r^O \) (\( p_r^A, p_r^O \in \{1, 2, \ldots\} \)), respectively. If our two-stage solution is not optimal, then we can get that:

\[
2 \leq p_r^A \leq |S_i| - 1 \quad \text{and} \quad 1 \leq p_r^O \leq p_r^A - 1.
\]

Proof. 1) Firstly, we can assert that \( p_r^A \) is the maximum reusing number according to Algorithm 2, so \( p_r^O \leq p_r^A - 1 \). 2) If \( p_r^A = p_r^O \), according to Algorithm 2, our two-stage solution will be optimal, which is contrary to the assumption. So we have \( p_r^O \leq p_r^A - 1 \). 3) If \( p_r^A = |S_i| \), which means all the VNF of \( S_i \) could reuse the existing VNF instances, then our two-stage solution will be optimal. So \( p_r^A \leq |S_i| - 1 \). 4) If \( p_r^A = 1 \), then \( p_r^A > p_r^O = 0 \), i.e., there’s no VNF could reuse the existing VNF instances in the optimal solution. Thus our two-stage solution is superior to the optimal solution, which also contradicts the assumption. So we have \( p_r^A \geq 2 \). All in all, we prove that \( 2 \leq p_r^A \leq |S_i| - 1 \) and \( 1 \leq p_r^O \leq p_r^A - 1 \).

Theorem 1. For service chain \( S_i \), we define \( R_{i, \text{max}} \) = \( \max\{R_{i_1}, R_{i_2}, \ldots, R_{i_{|S_i|}}\} \), \( R_{i, \text{min}} = \min\{R_{i_1}, R_{i_2}, \ldots, R_{i_{|S_i|}}\} \) and \( \gamma_i = R_{i, \text{max}} / R_{i, \text{min}} \). Assume that the server resource consumption and the length of the selected pre-routing path of our two-stage solution and the optimal solution are \( C_s^A \) and \( k^A \), and \( C_s^O \) and \( k^O \), respectively. The corresponding resource consumption of the two solutions are \( C_s^A + r_i \cdot k^A \) and \( C_s^O + r_i \cdot k^O \), respectively. Then, the worst-case performance bound \( f \) satisfies:

\[
f_i = \frac{C_s^A}{C_s^O} \leq \gamma_i. \tag{15}
\]

Proof. We proof Theorem 1 in three cases:

Case 1: If our two-stage solution is optimal, then \( C_s^A = C_s^O \), we have \( f_i = 1 \leq \gamma_i \).

Case 2: If our two-stage solution and the optimal solution are different \( (C_s^A > C_s^O) \) and yield from the same pre-routing path \( p_k \) whose length is \( k \). Then the bandwidth consumption is the same for our two-stage solution and the optimal solution: \( r_i \cdot k^A = r_i \cdot k^O = r_i \cdot k \). Hence, we have \( C_s^A > C_s^O \). By applying Lemma 1, we have \( p_r^O \leq p_r^A - 1 \).

Therefore, the server resource consumption of our two-stage solution and the optimal solution are \( C_s^A = R_x^s + \ldots + R_y^s \) and \( C_s^O = \frac{|S_i| - p_r^O}{\gamma_i} \), respectively. Furthermore, since \( C_s^A > C_s^O \) and \( r_i \cdot k^A = r_i \cdot k^O \), we have

\[
f_i = \frac{C_s^A}{C_s^O} = \frac{C_s^A + r_i \cdot k^A}{C_s^O + r_i \cdot k^O} < \frac{C_s^A}{C_s^O} = \frac{R_x^s + \ldots + R_y^s}{R_x^s + \ldots + R_y^s} \leq \frac{|S_i| - p_r^A}{|S_i| - p_r^O} \tag{16}
\]

The inequality (16) holds since we can easily obtain that \( \frac{b+c}{a+c} \leq \frac{b}{a} \), \( \forall a, b, c > 0 \) and \( b > a \).

In order to get the maximum \( f \), we calculate the maximum value of the numerator and the minimum value of the denominator respectively. Hence, we have:

\[
\begin{align*}
|S_i| - p_r^A & \leq |S_i| - p_r^O \\
& \leq \frac{R_{i, \text{max}}(|S_i| - p_r^A)}{|S_i| + 1 - p_r^A} = \gamma_i (1 - \frac{1}{|S_i| + 1 - p_r^A}) < \gamma_i.
\end{align*}
\]

(17)
Case 3: If our two-stage solution and the optimal solution are different (\(C^A > C^O\)) and yield from different pre-routing paths, let’s say \(p^A\) and \(p^O\) respectively. For path \(p^O\), we define the total resource consumption yields from our two-stage solution is \(C^O\). Since \(C^O\) and \(C^A\) are both from our solution, it is known that \(C^A \leq C^O\). Also because \(C^O\) is the optimal solution, then we have \(C^O \leq C^A \leq C^O\). It has been proven that \(f_i = \frac{C^O}{O_i} < \gamma_i\) in Case 2, hence, \(\frac{C^A}{O_i} \leq \frac{C^O}{O_i} < \gamma_i\).

All in all, for service chain \(S_i\), we have proven that the worst-case performance bound is \(f_i = \frac{C^A}{O_i} \leq \gamma_i\).

Corollary 1. Represent the worst-case performance bound \(F\) of all the arriving service chain \(S_1, S_2...S_N\) and assume \(\gamma_{max} = \frac{N}{O_1 \cdot \min} = \max\{\gamma_1, \gamma_2..., \gamma_N\}\), then we can derive \(F\) as follow:

\[
F = \frac{\sum_{i=1}^{N} C_i^A}{\sum_{i=1}^{N} C_i^O} \leq \frac{1}{\gamma_{max}} \cdot \frac{\sum_{i=1}^{N} C_i^O}{\sum_{i=1}^{N} C_i^O} = \gamma_{max}^k. \tag{18}
\]

The inequality (18) holds since \(C_i^A \leq \gamma_i \cdot C_i^O\) according to Theorem 1.

F. Complexity Analysis

Now we give a brief complexity analysis of our proposed algorithm. In Algorithm 1, without regard to branch reduction, finding the feasible paths in the \(N\)-node network requires \(O(N!)\) (from line 3-27) computation. In the second Algorithm 2, assigning a service chain with \(M\) nodes along a path requires \(O(2^M)\) (line 5-29). Then the time complexity of the two-stage algorithm is \(O(N! \cdot 2^M)\). However, since at the realistic network edge, to meet the low-latency requirement and the bandwidth constraints of the physical links, the computation complexity of the first algorithm will be significantly reduced. Besides, the length of the service chain is less than 10 [36] in the vast majority of cases, the computation complexity in Algorithm 2 is not large. We also conduct simulations to prove the two-stage algorithm is quite efficient with realistic network topologies, the results of the execution time is depicted as Fig. 5 and Fig. 6 in Sec. V.

V. EVALUATION

In this section, we conduct simulations to evaluate the performance of our proposed algorithm on top of the real-world network topologies and generate other parameters according to some previous works [37], [26].

Network topology. We implement the experiments on several network topologies of different sizes from the Internet topology zoo [38]: such as Bellsouth (51 nodes and 66 links), Cogentco (197 nodes and 245 links) and Kdl (754 nodes and 899 links) to simulate three different network scales. According to [37], we configure the nodes in the network with [0, 200] units of residual available resource capacities. To characterize the current configuration of each server, we randomly generate a set of reusable VNFs consisting of 0 to 8 VNFs picked from 20 VNF types. We further assign a random bandwidth capacity ranging from 0 to 1000 Mbps to each link between the servers.

Service chains. As aforementioned in Sec. IV, due to the assumption that the demand of service arrives into the system one by one, we build a single service chain at a time with a set of APs (source nodes) and a single destination randomly chosen from the set of nodes of the network topology. Specifically, we configure each AP with a processing capacity ranging from 100 Mbps to 200 Mbps. In addition, we generate a service chain by randomly picking 1 to 6 VNFs from 20 VNF types, and each type of VNF instance requires [20, 50] units of server resources in terms of CPU and memory. The flow rate of the chain ranges from 30 Mbps to 60 Mbps and the deadline are randomly distributed in [30, 80] ms [26]. We repeat each experiment 20 times to eliminate contingency.

Algorithm benchmark. In the simulations, we compare our proposed algorithm with the other three heuristic algorithms. Since our algorithm consists of two parts, i.e., constrained depth-first search algorithm (CDFSA) and the path-based greedy algorithm (PGA), we use these two parts to combine with some previous heuristics, (e.g., shortest path heuristic and first fit heuristic) to generate the other three algorithms, the details are as follows.

- **Shortest Path Heuristic + First Fit Assignment (SHP+FF):** in the first stage, it finds the shortest path by shortest path heuristic (SPH), and in the second stage, it assigns the VNFs along the shortest path by the first fit algorithm.
- **Constrained Depth-First Search Algorithm + First Fit Assignment (CDFSA + FF):** it selects the feasible paths with CDFSA, and employs first fit (FF) algorithm to assign the VNFs along the path. In the end, it will calculate the optimal solution among all the chosen paths.
- **Shortest Path Heuristic + Path-based Greedy Algorithm (SHP+PGA):** it employs the SPH to select the shortest path, and uses PGA to assign the VNFs.

Execution time. From the previous analysis, the time complexity of our proposed algorithm is correlated with the number of the servers and the length of the service chains. We first conduct the experiments in six realistic networks topologies with different scales. As shown in Fig. 5, as the number of servers ranges from 29 to 754, the execution time increases accordingly. Specifically, in the small-scale network with less than 110 servers, it takes less than 1 s to get a solution for 100 service chains. In the large-scale network, such as the 754-node case, the execution time is 12.39 s, which is still acceptable. Fig. 6 shows how the length of
paths using PGA to reuse as many VNFs as possible, which decision. On the other hand, we assign the VNFs along the path heuristic, we select all the feasible paths to make a global algorithms. On one hand, in the first stage, unlike the shortest the superiority comes from the two-stage design of our proposed algorithm comparing with other heuristic algorithms also increases with the flow rate of the service chains.

The total resource consumption. Since our objective in Eq. (9) is to jointly minimize the server resource and bandwidth resource, similar to [18], we mainly consider the length of the service chains which affects the consumption of server resource and the flow rate of the service chain which affects the consumption of the bandwidth resource. First of all, in Fig. 7, we depict the impact relationship between the length and the rate of the service chain with network Bellsouth (51 nodes) and Cogentco (197 nodes), respectively. As expected, the total resource consumption not only increases with the length, but also increases with the flow rate of the service chains.

Next, we explore the total resource consumption of our proposed algorithm comparing with other heuristic algorithms with different lengths and flow rates of services chains, respectively, as shown in Fig. 8 and Fig. 9. It is obvious that the total resource consumption of all algorithms increases with the length and flow rate of the service chain. Compared with other algorithms, our proposed algorithm can consume averagely 19.7%, 8.9% and 23.9% less resource than SPH+FF, SPH+PGA, and CDFSA+FF on network Bellsouth, and correspondingly 20.7%, 2.5% and 25.6% on Network Cogentco. The superiority comes from the two-stage design of our algorithms. On one hand, in the first stage, unlike the shortest path heuristic, we select all the feasible paths to make a global decision. On the other hand, we assign the VNFs along the paths using PGA to reuse as many VNFs as possible, which outperforms the first fit algorithm.

It is worth noting that the result of the algorithm SPH+PGA is almost close to ours, probably because the path of the optimal scheme of our algorithm is usually the shortest path in the network. In addition, based on the comparison between each subfigure pair in Fig. 7, Fig. 8 and Fig. 9, we can find that when the length and the flow rate are the same, the total resource consumption on network Cogentco is almost larger than that of network Bellsouth. Probably because we randomly generate the source and destination for a service chain, the larger the network is, the longer the feasible path might be.

Average latency. We now depict the average latency by comparing different algorithms. As shown in Fig. 10(a) and Fig. 10(b), the average latency of service chains ranges from 40 ms to 55 ms. Note that the SPH+FF and SPH+PGA always keep the same pace and incur minimum average latency, because they both choose the shortest path to deploy service chains. However, as shown in Fig. 8 and Fig. 9, the other three algorithms consume more resources than our algorithm. Also, even our algorithm is not the best one in terms of latency, it can also meet the latency requirements of the service chains in the network edge.

VI. CONCLUSION

In this paper, we investigate the VNF chain deployment problem at the network edge, to simultaneously reuse the server resources and reduce the bandwidth consumption, while providing the latency guarantees. We first reveal the dilemma between the resource utilization of servers and links. Then we formalize the problem as an MILP model and devise a two-stage solution to solve it. In the first stage, we employ CDFSA to select all the pre-routing paths. Given these paths, in the second stage, we design PGA to assign VNFs with minimum resource consumption. Our solution is proven to be near-optimal with a theoretically proved worst-case performance bound. Finally, we conduct simulations on realistic network topologies. The results demonstrate our proposed algorithm outperforms three other heuristic algorithms.